

1 - 14 Gauss elimination

Solve the linear system given explicitly or by its augmented matrix.

$$1. \quad 4x - 6y = -11$$

$$-3x + 8y = 10$$

```
ClearAll["Global`*"]
```

```
Solve[4 x - 6 y == -11 && -3 x + 8 y == 10]
```

$$\left\{ \left\{ x \rightarrow -2, y \rightarrow \frac{1}{2} \right\} \right\}$$

The green cell above agrees with the text answer.

$$3. \quad x + y - z = 9, 8y + 6z = -6, -2x + 4y - 6z = 40$$

```
ClearAll["Global`*"]
```

```
Solve[x + y - z == 9 && 8 y + 6 z == -6 && -2 x + 4 y - 6 z == 40]
```

$$\left\{ \left\{ x \rightarrow 1, y \rightarrow 3, z \rightarrow -5 \right\} \right\}$$

The green cell above agrees with the text answer.

$$5. \quad \begin{pmatrix} 13 & 12 & -6 \\ -4 & 7 & -73 \\ 11 & -13 & 157 \end{pmatrix}$$

```
{{65., 60., -30.}, {-20., 35., -365.}, {55., -65., 785.}}
```

```
ClearAll["Global`*"]
```

```
Solve[
```

```
13 x + 12 y + -6 z == 0 && -4 x + 7 y - 73 z == 0 && 11 x - 13 y + 157 z == 0, {x, y}]
```

```
{x -> -6 z, y -> 7 z}]
```

$$A = \begin{pmatrix} 13 & 12 & -6 \\ -4 & 7 & -73 \\ 11 & -13 & 157 \end{pmatrix}$$

```
{{13, 12, -6}, {-4, 7, -73}, {11, -13, 157}}
```

```
RowReduce[A]
```

```
{{1, 0, 6}, {0, 1, -7}, {0, 0, 0}}
```

Above: It is seen from inspection that $x = 6$ and $y = -7$, the same values found in the text.

$$7. \begin{pmatrix} 2 & 4 & 1 & 0 \\ -1 & 1 & -2 & 0 \\ 4 & 0 & 6 & 0 \end{pmatrix}$$

```
{{14., 28., 7., 0.}, {-7., 7., -14., 0.}, {28., 0., 42., 0.}}
```

```
ClearAll["Global`*"]
```

$$A = \begin{pmatrix} 2 & 4 & 1 & 0 \\ -1 & 1 & -2 & 0 \\ 4 & 0 & 6 & 0 \end{pmatrix}$$

```
{{2, 4, 1, 0}, {-1, 1, -2, 0}, {4, 0, 6, 0}}
```

```
e1 = RowReduce[A] // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & \frac{3}{2} & 0 \\ 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```
Solve[x +  $\frac{3}{2}$  z == 0 && y -  $\frac{1}{2}$  z == 0, {x, z}]
```

```
{{x → -3 y, z → 2 y}}
```

Above: The text sets $y = t$, but this is not necessary. Either one is arbitrary in value. To accord with text, $x = -3t$, $z = 2t$.

$$9. \begin{pmatrix} 0 & -2 & -2 & -8 \\ 3 & 4 & -5 & 13 \end{pmatrix}$$

```
{{0., -18., -18., -72.}, {27., 36., -45., 117.}}
```

```
ClearAll["Global`*"]
```

```
Solve[-2 y - 2 z == -8 && 3 x + 4 y - 5 z == 13, {x, y}]
```

```
{{x → -1 + 3 z, y → 4 - z}}
```

Above: The answer agrees with text, provided a parameter t is invented such that $z = t$.

$$11. \begin{pmatrix} 0 & 5 & 5 & -10 & 0 \\ 2 & -3 & -3 & 6 & 2 \\ 4 & 1 & 1 & -2 & 4 \end{pmatrix}$$

```
{{0., 55., 55., -110., 0.},  
{22., -33., -33., 66., 22.}, {44., 11., 11., -22., 44.}}
```

```
Clear["Global`*"]
```

```
A =  $\begin{pmatrix} 0 & 5 & 5 & -10 & 0 \\ 2 & -3 & -3 & 6 & 2 \\ 4 & 1 & 1 & -2 & 4 \end{pmatrix}$ 
{{0, 5, 5, -10, 0}, {2, -3, -3, 6, 2}, {4, 1, 1, -2, 4}}
```

Below: The row reduction comes up with a null row. Is that an ominous sign?

```
e1 = RowReduce[A]
{{1, 0, 0, 0, 1}, {0, 1, 1, -2, 0}, {0, 0, 0, 0, 0}}
```

The dot product has fewer equation seeds than there are rows in the A matrix. I play around with the assigned positions and signs of t1 and t2 to try to come up with the text answer.

```
e2 = e1.{x, y, z, t2, -t1}
{-t1 + x, -2 t2 + y + z, 0}
Solve[-t1 + x == 0 && -2 t2 + y + z == 0, {x, y}]
```

```
{{x -> t1, y -> 2 t2 - z}}
```

Let me trot out the text answer: $x = t_1$ arb.; $y = 2 t_2 - t_1$; $z = t_2$ arb. Rearranging the text answer results in the equation $y = 2z - x$. If I adopt the stance that $z = t_2$, then I can make the same equation out of the results above, that is, $y = 2z - x$.

```
13.  $\begin{pmatrix} 0 & 10 & 4 & -2 & -4 \\ -3 & -17 & 1 & 2 & 2 \\ 1 & 1 & 1 & 0 & 6 \\ 8 & -34 & 16 & -10 & 4 \end{pmatrix}$ 
```

```
{{0., 130., 52., -26., -52.}, {-39., -221., 13., 26., 26.},
{13., 13., 13., 0., 78.}, {104., -442., 208., -130., 52.}}
```

```
ClearAll["Global`*"]
```

```
e1 =  $\begin{pmatrix} 0 & 10 & 4 & -2 & -4 \\ -3 & -17 & 1 & 2 & 2 \\ 1 & 1 & 1 & 0 & 6 \\ 8 & -34 & 16 & -10 & 4 \end{pmatrix}$ 
{{0, 10, 4, -2, -4}, {-3, -17, 1, 2, 2},
{1, 1, 1, 0, 6}, {8, -34, 16, -10, 4}}
e2 = RowReduce[e1]
{{1, 0, 0, 0, 4}, {0, 1, 0, 0, 0}, {0, 0, 1, 0, 2}, {0, 0, 0, 1, 6}}
e3 = e2.{w, x, y, z, t1}
{4 t1 + w, x, 2 t1 + y, 6 t1 + z}
```

```
e4 = Solve[4 t1 + w == 0 && 2 t1 + y == 0 && 6 t1 + z == 0 && x == 0]
{{w -> -4 t1, x -> 0, y -> -2 t1, z -> -6 t1}}
```

```
e5 = e4 /. t1 -> -1
```

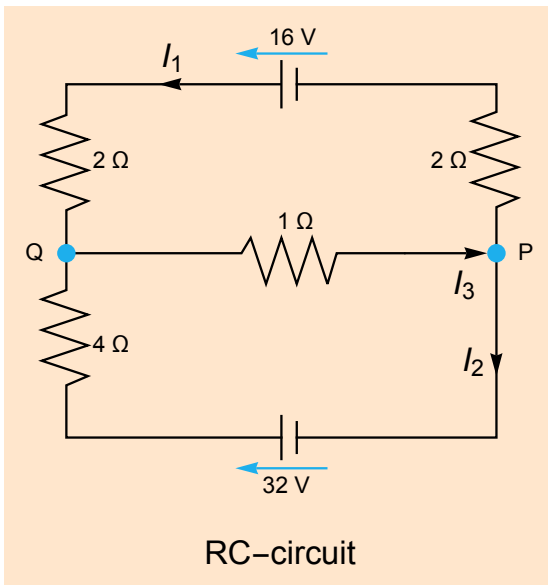
```
{{w -> 4, x -> 0, y -> 2, z -> 6}}
```

Above: The answer matches the text.

17 - 21 Models of networks

In problems 17 - 19, using Kirchhoff's laws (see example 2) and showing details, find the currents:

17. As shown in diagram below.



Considering only node P, and trying to put the Wikipedia information into effect, I have the equation $I_3 - I_2 - I_1 = 0$ (Kirchhoff's current law, which is strictly relative to P, not a clock) plus the two equations $-(1) * I_3 - 2 * I_1 - 16 - 2 I_1 = 0$ and $I_3 + 32 + 4 I_2 = 0$ (Kirchhoff's voltage law, in which each loop is considered with clockwise considered as positive).

```
In[81]:= Clear["Global`*"]
```

```
Solve[iI3 - iI2 - iI1 == 0 &&
  -(1) * iI3 - 2 * iI1 - 16 - 2 iI1 == 0 && iI3 + 32 + 4 iI2 == 0, {iI1, iI2, iI3}]
```

```
{{iI1 -> -2, iI2 -> -6, iI3 -> -8}}
```

The above green cell matches the text answer, the signs of current being uniformly opposite to the text answer merely informing me that all assumptions about current direction should

At this point specific values for the components can be set or reset.

```
In[84]:= mw = m1 /. {C1 -> 2, C2 -> 3, L -> 1, R -> 2}
```

$$\text{Out[84]=} \left(\begin{array}{cccccc|cccccc} 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & -1 & 0 \\ \hline 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right) S$$

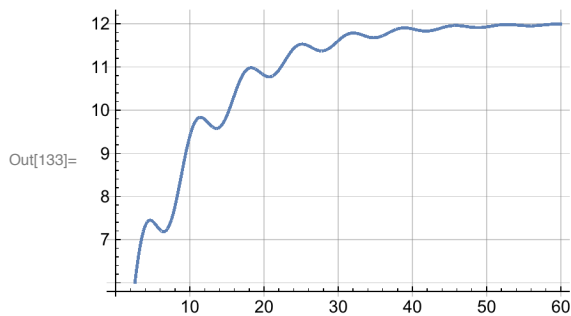
And finally the voltage is plugged in and the time-varying behavior is observed. I don't like the output response to this model, I would like to see multiple traces for the multiple loops (of the circuit).

```
In[130]:= opc = OutputResponse[{mw}, 12, {t, 0, 60}]
```

```
Out[130]= {InterpolatingFunction[ Domain{{0., 60.}} Output: scalar] [t]}
```

From the plot below, I seem to be seeing a gradual stabilization to the 12 volts entered as the 'e' value. However, I would expect to be seeing current, not potential.

```
In[133]:= Plot[opc, {t, 0, 60}, ImageSize -> 250, GridLines -> Automatic]
```



If state responses are considered, I see that there is a different one for each second. These are sinusoidal, but I don't know why the 5th is nearly flat, but the 4th and 6th are pretty curvy.

```
In[86]:= ops = StateResponse[{mw}, 12, {t, 0, 6}];
```

In[123]:=

```
p1 = Plot[ops[[1]], {t, 0, 5},  
  PlotStyle → Red, PlotRange → {{0, 5}, {-5, 10}}];  
p2 = Plot[ops[[2]], {t, 0, 5}, PlotStyle → Blue];  
p3 = Plot[ops[[3]], {t, 0, 5}, PlotStyle → Green];  
p4 = Plot[ops[[4]], {t, 0, 5}, PlotStyle → Orange];  
p5 = Plot[ops[[5]], {t, 0, 5}, PlotStyle → Pink];  
p6 = Plot[ops[[6]], {t, 0, 5}, PlotStyle → Black];
```

In[129]:= Show[p1, p2, p3, p4, p5, p6]

